

Assignment 5: Strategies for Calculating Antiderivatives and Definite Integrals Weeks 7 and 8

In class on Monday, [Week 7, Day 17]:

Finding an anti-derivative is the process of un-doing the derivative. That is, you look at an expression, and say to yourself, “Hm-mmm, this is the result of taking the derivative. What did I take the derivative of?” Sometimes you can simply “see” what the original expression was – but not usually. So we need some strategies for un-doing the process of differentiation. In Section 8.4, we will be learning three different strategies for finding antiderivatives.

Our study of the Fundamental Theorem of Calculus in Chapter 7 taught us several important ideas:

- 1) A definite integral can be associated with the *area* bounded by a graph and the x -axis.
 - 2) However, if the function is sometimes negative (so that the graph of the function goes below the x -axis, the so-called “area” calculated by evaluating a definite integral will sometimes be negative, and in some situations portions of the area will cancel each other out.
 - 3) Since area is essentially a product (height \times width), many problems that involve products can be expressed as definite integral. Thus, since $distance = velocity \times time$, the *net distance* traveled can be calculated by evaluating an appropriate definite integral.
 - 4) Evaluating a definite integral requires two steps – finding the antiderivative, and then performing an appropriate subtraction.
- Section 8.4.1 *Algebraic Substitutions*
Algebraic substitutions (or u -substitutions) can be used when the derivative was taken using the chain rule.
 - Homework:
 - Study Section 8.4.1, and attempt the Activities and Checkpoints.
 - Which of the problems in Exercises 1 – 2 can you do using an algebraic substitution? Do them.
 - Continue to work on Web Work problem sets. There are five Web Work problem sets which use methods from Section 7.2: *The Fundamental Theorem of Calculus*, (Chap7_A, B, C, D, and E). Attempt all five of these problem sets. To get credit for attempting these, your score should be at least 80% of the total points possible. If you don’t get a problem correct on the first attempt, try it again until you get it – or ask me about the problems that you find difficult. Most of these problems are drill-and-practice for calculating definite integrals, but each set has at least one problem that will make you think a little harder. These five problem sets are due on Wednesday morning, March 10, and are good practice problems for Test 2.

In class on Wednesday, [Day 18]:

- Section 8.4.2 *Trigonometric Substitutions*
Trigonometric substitutions are a little more complicated. Sometimes a trigonometric substitution is helpful when there is an expression that “reminds me of the Pythagorean Theorem.” Sometimes you need to draw on any additional knowledge you have of trigonometry. For example, study Example 2 in this section to see how a “double angle formula” can be used to simplify the antiderivative of Exercise 1c.
- Homework:
 - Study Section 8.4.2, and work through the Examples, Activities and Checkpoints.
 - Which of the problems in Exercises 1 – 2 can you do using a trigonometric substitution? Do them.
 - Do Exercise 7.

In class on Friday, [Day 19]:

- Section 8.4.3 *Integration by Parts*
Integration by parts is used to find the antiderivative when the derivative was calculated using the product rule.
As a general rule, we always try to solve a problem by the easiest method available to us. So when you look at an antiderivative problem:

- 1) Think about whether you can simply reverse a derivative rule that you know.
 - 2) Then, look for a u -substitution. In this case you will see a composite function (that is, a “nested function”), and the derivative of the “inside function” will be one of the terms in the product.
 - 3) If the antiderivative involves an expression like $\sqrt{a^2 + x^2}$ or $\sqrt{a^2 - x^2}$ or $\sqrt{x^2 + a^2}$ – and a simple u -substitution didn’t work, try a trigonometric substitution.
 - 4) If the antiderivative involves $\sin^2(x)$ or $\cos^2(x)$ (and a simple u -substitution doesn’t work) try to simplify the expression using a double-angle formula.
 - 5) If the expression for which you are looking for an antiderivative involves a product, think about whether you can see this as a *product of one function and the derivative of another function*. In this case, integration by parts might work.
- Homework:
 - Study Section 8.4.3, and work through the Examples, Activities and Checkpoints.
 - By this time, you should be able to do all of the parts of Exercises 1 and 2.
 - Do Exercise 8.

Test 2 is scheduled for Wednesday, March 10. Test 2 will cover the material that was on Test 1, PLUS material from Chapter 8, Sections 8.1, 8.3, and 8.4. Doing better on Test 2 than you did on Test 1 will improve your grade in this course. The study Guide for Test 2 will be available after class on Friday, March 5.

In class on Monday, [Week 8, Day 20]:

- Review for the test

In class on Wednesday, [Day 21]:

- Test 2, with Benchmark 2