Assignment 2: Antidifferentiation
Week 2

In class on Wednesday, [Day 4]:

- More work on Section 6.2: *Separation of Variables*
  - We will work through the Maple worksheets for Activities 1, 2, and 3 in Section 6.2. Maple is now available on the computers in BH 056, and will soon be available in other labs on campus.
  - Then we will go through methods that can be used to solve the natural growth problem (and other similar differential equations) symbolically that are presented in Section 6.2.
- Complete the Activities and Checkpoints of Section 6.2, being sure that you understand what is going on.
- Continue your work on the Web Work [Note: We may need to update this URL.] (http://webwork.maa.org/webwork2/) problems that have been assigned. You may work together on these problems, but to get credit each of you must log into Web Work under your own name. Do at least four of the six problem sets for Section 5.6 (A – F).

In class on Friday, [Day 5]:

- Section 6.3: *The Logistic Growth Equation*
  - As we have seen in Chapter 2 and in Section 6.2, the model for natural growth describes population growth for a while, and then environmental constraints (space, food, and other resources necessary for growth) slow the growth. As the population gets large, logistic growth is a better model. Section 6.2 introduces the logistic growth model, and Section 6.3 develops methods that we can use to solve this model.
  - This is a long section, and we will spend both Friday and Monday working through the techniques presented in this Section. The project is an application is a direct application of these methods. You will have to study Section 6.3 carefully to complete the project.
- Project 1: *Sky Diving*
  - Exercise 5 from Section 5.1 begins, "Pilots and parachutists know (and physicists confirm) that the Velocity-Squared Model applies to the resisting force of air on an airplane wing or a falling human body, with or without a parachute." Project 1, given at the end of Chapter 6, leads you through a step-by-step process to solve the differential equation which appears in Exercise 5, Section 5.1. This is a challenging problem that builds on the Velocity-Squared Model. Review the set up for the velocity-squared model in Section 5.1 and study Section 6.3 very carefully to see a method that can be used to solve this differential equation.
  - Project 1 is DUE on Friday, February 5.
  - The Grading Rubric for Project 1 is available as a separate handout.

In class on Monday, [Day 6]:

- More on Section 6.3: *The Logistic Growth Equation*
- Questions and discussion about Project 1

In class on Wednesday, [Day 7]:

- We will begin Chapter 7: *The Fundamental Theorem of Calculus*

Teaching Notes for Day 6: Families of Differential Equations

In population growth and interest-rate problems, we found that a function representing the quantity of a substance present at time t had the form Ae^{kt}, an exponential with a positive exponent, and its values therefore increased without bound as t became large (as long as there is room for growth or we don't make any withdrawals for the savings account).

$$\frac{dy}{dt} = ky \longrightarrow y(t) = Ae^{kt} + C$$

In radioactive decay problems, we found that a function representing the quantity of a substance present at time t had the form Ae^{kt}, an exponential with a negative exponent, and its values therefore decreased to zero as t became large.

$$\frac{dy}{dt} = ky \longrightarrow y(t) = Ae^{-kt} + C$$

With Newton's Law of Cooling, the solution function representing temperature had two terms: a constant, which was the ambient temperature, and an exponential that resembled radioactive decay. Thus, as time became large, the exponential term decreased to zero, and the temperature decreased to the ambient temperature.

$$\frac{dT}{dt} = -k(T - A) \longrightarrow T(t) = A + Ce^{-kt}$$

For falling bodies with air resistance proportional to velocity, the computation was similar to Newton's Law of Cooling, but the factor multiplying the exponential term was negative. Thus the solution for velocity as a function of time

$$\frac{dv}{dt} = g - kv \longrightarrow v(t) = \sqrt{\frac{g}{c}}\,\frac{1 - \exp\left(-2\sqrt{gct}\right)}{1 + \exp\left(-2\sqrt{gct}\right)}$$

With the logistic model, we see a similar phenomenon, an approach as time goes on toward a stable population level, namely, the maximum supportable population M.

$$\frac{dP}{dt} = kP(M - P) \longrightarrow P(t) = \frac{P_0 M e^{Mkt}}{M - P_0 + P_0 e^{Mkt}}$$

In the project, we are again faced with solving a differential equation:

$$\frac{dv}{dt} = g - cv^2 \longrightarrow v(t) = ?$$

What is a strategy for solving these differential equations?

1. Separate the variables
2. Find antiderivatives on each side
3. Solve for the dependent variable as a function of the independent variable